1. MULTIPLYING BINOMIALS

A *binomial* is an algebraic expression which is the sum or the difference of two terms.

We wish to understand the product of two binomials; that is, we wish to compute $(a + b)(c + d)$. We use the distributive property to multiply this. Let $x = (c + d)$. Then

$$
\begin{aligned}
(a + b)(c + d) &= (a + b)x \\
&= ax + bx &&\text{using right distribution} \\
&= a(c + d) + b(c + d) \\
&= ac + ad + bc + bd
\end{aligned}
$$

The mnemonic FOIL (First, Outer, Inner, Last) gives an order to how we find these last four terms. Use this technique to compute the following facts.

Example 1. Expand $(x + 2y)(3z + 4)$.

Solution. I don't like the acronym FOIL (it is the distributive property), but let us go over how to use it anyway. The first terms are x and $3z$, the outer terms are x and 4, the inner terms are $2y$ and $3z$, and the last terms are $2y$ and 4.

- First: $\qquad x \times 3z = 3xz$
- Outer: $\qquad x \times 4 = 4x$
- Inner: $\qquad 2y \times 3z = 6yz$
- Last: $\qquad 2y \times 4 = 8y$

Add these up to get

$$(x + 2y)(3z + 4) = 3xz + 4x + 6yz + 8y.$$

$\square$

Example 2. Expand $(a + b)^2$.

Solution. We have

$$(a + b)^2 = (a + b)(a + b) = aa + ab + ba + bb = a^2 + 2ab + b^2.$$

$\square$

Example 3. Expand $(x - 7)^2$.

Solution. Apply the formula above with $a = x$ and $b = -7$. Then

$$(x - 7)^2 = x^2 - 14x + 49.$$

$\square$

Example 4. Expand $(x - r)(x - s)$.

Solution. We have

$$(x - r)(x - s) = x^2 - xs - rx + rs = x^2 - (r + s)x + rs.$$

$\square$

2. Squaring Binomials

Every student should memorize these facts.

Proposition 1. *Let a and b be numbers. Then*
- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$

Proposition 2. *Let x be a variable. Then*

- $(x - 1)^2 = x^2 - 2x + 1$
- $(x - 2)^2 = x^2 - 4x + 4$
- $(x - 3)^2 = x^2 - 6x + 9$
- $(x - 4)^2 = x^2 - 8x + 16$
- $(x - 5)^2 = x^2 - 10x + 25$
- $(x - 6)^2 = x^2 - 12x + 36$
- $(x - 7)^2 = x^2 - 14x + 49$
- $(x - 8)^2 = x^2 - 16x + 64$
- $(x - 9)^2 = x^2 - 18x + 81$

- $(x + 1)^2 = x^2 + 2x + 1$
- $(x + 2)^2 = x^2 + 4x + 4$
- $(x + 3)^2 = x^2 + 6x + 9$
- $(x + 4)^2 = x^2 + 8x + 16$
- $(x + 5)^2 = x^2 + 10x + 25$
- $(x + 6)^2 = x^2 + 12x + 36$
- $(x + 7)^2 = x^2 + 14x + 49$
- $(x + 8)^2 = x^2 + 16x + 64$
- $(x + 9)^2 = x^2 + 18x + 81$